



Language: English

Day: 2

Wednesday, April 13, 2016

Problem 4. Two circles, ω_1 and ω_2 , of equal radius intersect at different points X_1 and X_2 . Consider a circle ω externally tangent to ω_1 at a point T_1 , and internally tangent to ω_2 at a point T_2 . Prove that lines X_1T_1 and X_2T_2 intersect at a point lying on ω .

Problem 5. Let k and n be integers such that $k \geq 2$ and $k \leq n \leq 2k - 1$. Place rectangular tiles, each of size $1 \times k$ or $k \times 1$, on an $n \times n$ chessboard so that each tile covers exactly k cells, and no two tiles overlap. Do this until no further tile can be placed in this way. For each such k and n , determine the minimum number of tiles that such an arrangement may contain.

Problem 6. Let S be the set of all positive integers n such that n^4 has a divisor in the range $n^2 + 1, n^2 + 2, \dots, n^2 + 2n$. Prove that there are infinitely many elements of S of each of the forms $7m, 7m + 1, 7m + 2, 7m + 5, 7m + 6$ and no elements of S of the form $7m + 3$ or $7m + 4$, where m is an integer.