language: English

4^{th} Middle European Mathematical Olympiad

INDIVIDUAL COMPETITION
11^{th} SEPTEMBER, 2010

Problem I-1.
Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$, we have

$$f(x+y) + f(x)f(y) = f(xy) + (y+1)f(x) + (x+1)f(y).$$

Problem I-2.
All positive divisors of a positive integer N are written on a blackboard. Two players A and B play the following game taking alternate moves. In the first move, the player A erases N. If the last erased number is d, then the next player erases either a divisor of d or a multiple of d. The player who cannot make a move loses. Determine all numbers N for which A can win independently of the moves of B.

Problem I-3.
We are given a cyclic quadrilateral $ABCD$ with a point E on the diagonal AC such that $AD = AE$ and $CB = CE$. Let M be the center of the circumcircle k of the triangle BDE. The circle k intersects the line AC in the points E and F. Prove that the lines FM, AD, and BC meet at one point.

Problem I-4.
Find all positive integers n which satisfy the following two conditions:

(i) n has at least four different positive divisors;

(ii) for any divisors a and b of n satisfying $1 < a < b < n$, the number $b - a$ divides n.

Time: 5 hours
Time for questions: 45 min
Each problem is worth 8 points.
The order of the problems does not depend on their difficulty.