

**Problem T-1**

Determine all triples  $(a, b, c)$  of real numbers satisfying the system of equations

$$\begin{aligned}a^2 + ab + c &= 0, \\b^2 + bc + a &= 0, \\c^2 + ca + b &= 0.\end{aligned}$$

**Problem T-2**

Let  $\mathbb{R}$  denote the set of real numbers. Determine all functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x)f(y) = xf(f(y-x)) + xf(2x) + f(x^2)$$

holds for all real numbers  $x$  and  $y$ .

**Problem T-3**

A tract of land in the shape of an  $8 \times 8$  square, whose sides are oriented north–south and east–west, consists of 64 smaller  $1 \times 1$  square plots. There can be at most one house on each of the individual plots. A house can only occupy a single  $1 \times 1$  square plot.

A house is said to be *blocked from sunlight* if there are three houses on the plots immediately to its east, west and south.

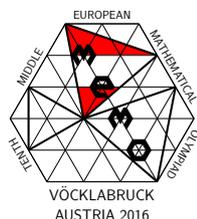
What is the maximum number of houses that can simultaneously exist, such that none of them is blocked from sunlight?

Remark: By definition, houses on the east, west and south borders are never blocked from sunlight.

**Problem T-4**

A class of high school students wrote a test. Every question was graded as either 1 point for a correct answer or 0 points otherwise. It is known that each question was answered correctly by at least one student and the students did not all achieve the same total score.

Prove that there was a question on the test with the following property: The students who answered the question correctly got a higher average test score than those who did not.



**Problem T–5**

Let  $ABC$  be an acute-angled triangle with  $AB \neq AC$ , and let  $O$  be its circumcentre. The line  $AO$  intersects the circumcircle  $\omega$  of  $ABC$  a second time in point  $D$ , and the line  $BC$  in point  $E$ . The circumcircle of  $CDE$  intersects the line  $CA$  a second time in point  $P$ . The line  $PE$  intersects the line  $AB$  in point  $Q$ . The line through  $O$  parallel to  $PE$  intersects the altitude of the triangle  $ABC$  that passes through  $A$  in point  $F$ .

Prove that  $FP = FQ$ .

**Problem T–6**

Let  $ABC$  be a triangle with  $AB \neq AC$ . The points  $K, L, M$  are the midpoints of the sides  $BC, CA, AB$ , respectively. The inscribed circle of  $ABC$  with centre  $I$  touches the side  $BC$  at point  $D$ . The line  $g$ , which passes through the midpoint of segment  $ID$  and is perpendicular to  $IK$ , intersects the line  $LM$  at point  $P$ .

Prove that  $\sphericalangle PIA = 90^\circ$ .

**Problem T–7**

A positive integer  $n$  is called a *Mozartian number* if the numbers  $1, 2, \dots, n$  together contain an even number of each digit (in base 10).

Prove:

- (a) All Mozartian numbers are even.
- (b) There are infinitely many Mozartian numbers.

**Problem T–8**

We consider the equation  $a^2 + b^2 + c^2 + n = abc$ , where  $a, b, c$  are positive integers.

Prove:

- (a) There are no solutions  $(a, b, c)$  for  $n = 2017$ .
- (b) For  $n = 2016$ ,  $a$  must be divisible by 3 for every solution  $(a, b, c)$ .
- (c) The equation has infinitely many solutions  $(a, b, c)$  for  $n = 2016$ .